**Differential Geometry** (Fall of 2017, for undergraduates, the following [1] will be the textbook; Tuesday and Thursday, 10:00-11:40, Eastern Top Teaching Building 212)**:**

This is a course for undergraduates. It mainly  concerns the geometry of curves and surfaces in Euclidean spaces, especially 3-space**.** We mainly concern local aspects but also some global aspects of surfaces.

To study the global aspects of surfaces, we‘ll try to introduce the notions of abstract surfaces---2-dim manifoldsand 2-riemannian manifolds**;** and in turn we‘ll introduce some general notions of riemannian geometry (but restricted to the 2-dim case)**:** geodesic, exponential map, completeness, Jacobi fields, conjugate points, and comparison theorems (if time admitted) etc.  Futhermore, we‘ll informally introduce topological classification of closed orientable surfaces by nonnegative integers---genus---by means of tirangulation. Then, we‘ll prove the famous Gauss-Bonnet formulae.

**Some preliminaries:**topology of Euclidean space; tangent space and tangent bundle, differential (tangent map) of a differentiable map; local behaviour of differentiable map (inverse function and implicit function theorems)

**Chap. 1** Curves in Euclidean 3-spaces

1. (regular) parametrized curves, arc length parameter; tangent vector, normal and binormal vectors, osculating plane, normal plane and rectifying plane

2. Frenet frame and Frenet formulae, curvature and torsion; canonical (normal) form near a point of curves; geometric implication of curvature and torsion; plane curves

3. fundamental theorem for curves in 3-space (uniqueness and existence to a curve with arc length parameter in 3-space with prescribed curvature (>0) and torsion)

**Exercises:** 1. compute the curvature and torsion of a curve under general regular parameters;

2. think why "curvature" and "torsion" are (geometric) invariants of a space curve---independent of choice of parameters;

3. derive the canonical (normal) form at a point of a 3-space curve and show the geometric meaning of curvature and torsion;

4. use the normal form of a curve to understand Corollary 1.5.4 and draw the projections in the corresponding planes;

5. finish Ex. 1.6.4.

**\*Some additional readings** for Chap. 1 (some global aspects of plane curves):

1. Chap. 2 of the textbook;

2. (general) 4 vertex theorem and its converse ([1] D. DeTurck, H. Gluck, D. Pomerleano, and D. Shea Vick, The four vertex theorem and its converse, Notices of AMS, Vol. 54, No. 2, 192-207; [2] Bjoern E. J. Dahlberg, The converse of the four vertex theorem, Proc. AMS, Vol. 133, No. 7, 2131-2135) .

**Chap. 2** (Parametrized) surfaces in Euclidean 3-spaces--local theory

1. (regular) parametrized surfaces, tangent space (tangent vectors), changes of variables of surfaces, unparametrized surfaces; vector fields along surfaces: tangential (normal) vector fields, coordinate vector fields; unit normal vector field of surface---Gauss map, Gauss frame; **differentials of composed maps (the special case of changes of variables)**

2. the 1st fundamental form: independent of parameters (so it is the geometric invariant of the corresponding unparametrized surface)

3. the 2nd fundamental form: independent of parameters (so it is the geometric invariant of the corresponding unparametrized surface); Weingarten map (transformation) of tangent spaces of surfaces; examples.

4. curves on surfaces: line element, Meusnier‘s theorem, normal curvature

5. Weingarten map: principal curvature, Gauss curvature, mean curvature

6. canonical form of a surface at a point: elliptic, parabolic, and hyperbolic points; vector field and its trajectories and first integral; coordinate system generated by two vector fields which are linearly independent at some points: orthogonal coordinate systems; principal directions and (equation of) lines of curvature, Rodriques‘ Theorem, principal curvature coordinate system (coordinate system of curvature lines); asymptotic directions and (equation of) asymptotic curves, coordinate system of asymptotic curves

7. ruled surfaces and developable surfaces: classification of developable surfaces

8. equations of motion for surfaces and structure equations (compatibility equations): Gauss‘s theorema egregium; fundamental theorem for surfaces in Euclidean 3-space

9. Gauss map and geometric explanation of Gauss curvature: geometry of second fundamental form is equivalent to geometry of Gauss map; minimal surfaces: critical points of area functional

10. surfaces of revolution with constant Gauss curvature (pseudo-sphere) and zero mean curvature (catenary and catenoid) (Ex.)

**Exercises**: 1. Ex.4,7,8 of Section 2-5 in [2];

2. Prove the remark in Page 45;

3. Prove 3.9.1, 3.9.2, 3.9.3, 3.9.4, 3.9.6, 3.9.7, 3.9.8\*(5.7.4);

4. write the Gauss‘ equation under orthogonal coordinates;

5. write Mainardi-Codazzi equations under principal directions coordinate systems (parameter net of lines of curvature)

**Chap. 3** Intrinsic geometry of surfaces in Euclidean 3-space--local theory

1. vector fields and covariant differentiation; parallel translation

2. geodesic curvature, Liouville formula; geodesics

3. Gauss-Bonnet formula for simple closed domains with piece-wise smooth boundary in a surface

4. exponential map, geodesic polar coordinate, Gauss lemma, (local) minimality of geodesics; surfaces of constant curvature

**Exercises**: 1.

**Chap.4** Selected topics of 2-dimensional Riemannian geometry

**References**

[1]  W. Klingenberg: **A courses in Differential Geometry**, Springer-Verlag

[2]  Manfredo do Carmo: **Differential Geometry of Curves & Surfaces**, revised & updated 2nd edition, Dover

[3]  彭家贵，陈卿: **微分几何**， 高等教育出版社