

# Graph Theory and Network (2014-2015-2)

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Class Room: Zhongyuan (中院) 104

Monday 10:00–11:40 and

Wednesday 10:00–11:40

Roughly speaking, the course will consists of the three parts.

- (i) Introduction to general graph theory,
- (ii) Algebraic graph theory,
- (iii) Network theory from the graph theoretical viewpoint.

There are many books on this subject. I will not use a single text book, but as a reference book for part (i), I recommend the book Introduction to Graph Theory (5th edition) by Robin Wilson, Person Education Limited 2010. The reference books for part (ii) and part (iii) will be announced later in the class.

Syllabus (tentative and subject to change):

1. Definitions and Examples of Graphs and Digraphs
  2. Paths and Cycles: Connectivity of graphs, Eulerian and Hamiltonian graphs
  3. Trees: Fundamental properties of trees and spanning trees
  4. Cycles and Cuts
  5. Adjacency matrix of a graph and spectrum of a graph
  6. Laplacian of a graph
  7. Matrix-Tree Theorem
  8. More on algebraic graph theory (strongly regular graphs and related topics)
  9. Planarity of Graphs
  10. Coloring of Graphs
  11. Flows in Networks
  12. Matchings and Hall's Marriage Theorem
  13. Network Flows and Mini-Max Theorem
  14. Menger's Theorem
  15. Electric Networks
  16. Linear Algebraic Method to study Networks
  17. Group Theory and graph theory
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## Homework.

1. Determine all graphs of 5 vertices.
2. Determine all regular graphs of valency 3 with 6 vertices.
3. Determine all regular graphs of valency 3 with 7 vertices.
4. (slightly difficult) Determine all regular graphs of valency 3 with 8 vertices.
5. Determine all regular graphs of valency 3 with 10 vertices such that there are no triangles and no quadrangles.
6. (Extremely difficult. A famous open problem.) Determine all regular graphs of valency 57 with 3250 vertices such that there are no triangles and no quadrangles.
7. Let  $Q_k$  be the  $k$ -cube. Prove that  $Q_k$  is a bipartite regular graph of valency  $k$ .