

《概率论》课程大纲

一. 课程简介

课程名称：概率论

学时：48

先修课程：线性代数（必须），离散数学（推荐）

考核方式：60%平时成绩（课后作业），40%期末考试

二. 教学内容与授课进度

课程内容分为以下几个部分：概率空间、随机变量与分布、数学证明的概率法、概率集中现象、马尔可夫链与随机过程。具体的内容与课时安排如下：

• **Probability Space: 4课时**

- probability events: 2课时
 - classic probability
 - combinatorial enumeration
 - Kolmogorov's axioms for probability
 - inclusion-exclusion & Boole's inequality
- conditional probability and independence: 2课时
 - law of total probability
 - Bayes law
 - law of successive conditioning
 - Schwartz-Zippel theorem

• **Random Variables: 8课时**

- discrete random variables: 2课时
 - random variables
 - distributions of discrete random variables
 - expectation, linearity of expectation
 - entropy, Jensen's equality
- balls and bins: 2课时
 - birthday problem
 - coupon collector
 - stable marriage
 - occupancy problem
- general random variables: 4课时
 - continuous random variables
 - probability density function
 - joint distributions
 - function of random variables
 - variance and covariance
 - correlation
- moment and deviation: 4课时
 - variance and moment

- Markov's inequality
- Chebyshev's inequality
- threshold phenomenon in Erdos-Renyi random graph
- 2-point sampling
- **The probabilistic method: 8课时**
 - basic methods: 4课时
 - counting argument
 - averaging principle
 - alternations
 - double counting: Sperner's theorem, Erdos-Ko-Rado
 - advanced methods: 4课时
 - min-cut
 - conditional probability method
 - Lovasz local lemma
- **Concentration: 12课时**
 - concentration in the limit: 2课时
 - law of large numbers (LLN)
 - central limit theorem (CLT)
 - Le Cam's theorem
 - Chernoff bound: 2课时
 - moment generating function
 - Bernstein's proof of Chernoff-Hoeffding bound
 - Chernoff bound applications: 2课时
 - parameter estimation
 - set balancing
 - counting DNF
 - martingales: 2课时
 - martingales
 - edge exposure martingales
 - stopping times
 - Wald's equation
 - concentration of measures: 4课时
 - Azuma's inequality
 - Doob sequence
 - method of bounded differences
 - Johnson-Lindenstrauss lemma
- **Markov Chain: 12课时**
 - basics & hitting/covering: 4课时
 - Markov chain and random walk
 - ergodicity
 - fundamental theorem of Markov chain
 - hitting time and electric network
 - cover time
 - mixing time and coupling: 4课时
 - total variation distance
 - mixing time
 - coupling of random variables
 - coupling lemma
 - card shuffling
 - sample graph colorings
 - expander graphs and conductance: 4课时

- spectral approach of mixing time
- expander graph
- graph spectrum
- Cheeger's inequality
- conductance and canonical path
- sample matchings

三. 教材和参考书

教材：

- Michael Mitzenmacher and Eli Upfal. *Probability and Computing*. Cambridge University press, 2005.
- William Feller. *An Introduction to Probability Theory and Its Applications, Vol. 1, 3rd Edition*. Wiley, 1968.

参考书：

- Dimitri Bertsekas and John Tsitsiklis. *Introduction to Probability, 2nd Edition*. Athena Scientific, 2008.
- Kai Lai Chung. *A Course in Probability Theory, 3rd Edition*. Academic Press, 2000.
- E. T. Jaynes and G. Larry Bretthorst. *Probability Theory: The Logic of Science*. Cambridge University Press, 2003.

参考在线讲义：

- Alistair Sinclair. *Randomness and Computation*. UC Berkeley.
<http://www.cs.berkeley.edu/~sinclair/cs271/f11.html>
- Alistair Sinclair. *Markov Chain Monte Carlo: Foundations and Applications*. UC Berkeley.
<http://www.cs.berkeley.edu/~sinclair/cs294/f09.html>
- Constantinos Daskalakis. *Probability and Computation*. MIT.
<http://people.csail.mit.edu/costis/6896sp11/>