Homework 10

Mathematics in Computer Science

1. What are the Boolean functions

   (a) $(x \to y) \to y$?

   (b) $(x \to y) \to x$?

2. There are 16 Boolean functions of two variables. A subset of the 16 Boolean functions is said to be *universal* if the other functions can be expressed in terms of functions in the subset. A subset is *minimal* if deleting any element of the subset results in a non universal set. 'AND' and 'NOT' is a minimal universal set as is 'OR' and 'NOT'.

   Give three other such sets. Demonstrate that your sets are indeed universal.

3. How many Boolean functions of n variables?

4. There are two types of people in a certain group. Liars who always lie and truth tellers who always say the truth. In each of the following can you say anything about the type of each person?

   (a) A says one of us is a liar. B says nothing.

   (b) A says both of us are truth tellers and B says A is a liar.

   (c) A and B both say we are truth tellers.

5. Prove that $x \to y$ is equivalent to $\overline{y} \to \overline{x}$

6. Aces and eights - 8 cards 4 aces 4 eights, three players, six cards dealt, two to each player. Other two face down. Each layer show his or her hand to the other two players. In turn each player tries to determine his or her hand. If player does not know his or her hand he/she must say so.

   (a) Alice two aces, Bob two eights, you.
   Alice don't know, Bob don't know. Then what are you holding?

   (b) you, Alice two eights, Bob ace + eight.
   You don't know, Alice don't know, Bob don't know. Then what are you holding?