Homework 7

Mathematics in Computer Science

1. Prove that
$$\binom{m+n}{k} = \sum_{i=0}^{k} \binom{m}{k-i}\binom{n}{i}$$

2. Use the principle of inclusion and exclusion to determine the number of integers between 1 and 1000 not divisible by 2,3, 5, or 7.

3. If there are 13,000 undergraduate students at Cornell, how many must have the same birthday?

4. How many Boolean functions of n variables?

5. Prove that
$$\binom{m+n}{k} = \sum_{i=0}^{k} \binom{m}{k-i}\binom{n}{i}$$

6. How many ways can one write seven as the sum of four nonnegative integers?

7. (a) Role a k-sided dice three times. How many possible outcomes are there?
   (b) What is the probability of the same face (number) of the k-sided dice appearing exactly two times in the three roles?
   (c) If it were a 6-sided dice what is the probability of the same face appearing exactly two times?
   (d) What is the probability of the same face appearing three times?
   (e) What is the probability of the no face ever appearing twice?
   (f) What is the sum of the above three probabilities?

8. Given 100 boxes some of which have paint spots as listed below how many boxes have not paint spots?
   (a) 40 with some red paint
   (b) 60 with some blue paint
   (c) 10 with some black paint
   (d) 30 with some red and some blue
   (e) 5 with some red and some black
   (f) 5 with some blue and some black
   (g) 2 with some of all three colors

Think about the following question (Optional)

1. Let n be a positive integer. How many ways can one assign positive integers to $i_1, i_2, \ldots, i_k$ such that $\sum_{j=1}^{k} i_j = n$.