Homework 5

Mathematics in Computer Science

1. What is $800^{35} \pmod{11}$?
2. (a) if $i \geq 0$ what is $i \pmod{p}$?
   (b) if $i < 0$ what is $i \pmod{p}$?
3. Solve
$$\begin{aligned} 3x + 2y &= 1 \qquad \pmod{7} \\ x - y &= 5 \end{aligned}$$
4. (a) Use Euclead's algorithm to compute the gcd of 495 and 210. Write out the steps.
   (b) What is the prime factorization of 495 and of 210?
   (c) Is your answer to part (a) correct?
5. (Extended Eucleadean Algorithm) What is multiplicative inverse of 400 mod 997?
6. What is the relationship between $a \times b$, $\gcd(a, b)$, and lcm(a, b)?
7. I wish to calculate the value of a number of ten digits numbers such as 4756213912 mod 9. Prove that I can get the correct answer by adding the digits and taking the result mod 9.
8. Consider the integers mod p. Notice that certain numbers are perfect squares. For example, $3^2 = 2 \bmod p$.
   (a) What fraction of the numbers mod p are perfect squares? Hint: Explore integers modulo a small prime like 7 or 11.
   (b) Give a proof of your answer to a.
9. (a) Prove that $= \pmod{p}$ is an equivalence relation.
   (b) Note that in mod arithmetic one represents each equivalence class by a representative of the class and defines arithmetic for the representative elements. If $p = 3$ what are the addition and multiplication tables for the representative elements?

Think about the following question (Optional, no need for submission)

1. Your task is to design a "prove" problem, which needs to be proved by the induction proof method, but you have to use a stronger induction hypothesis (than the problem itself) during the induction step.