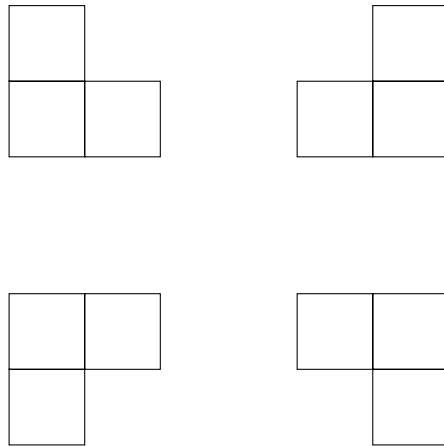


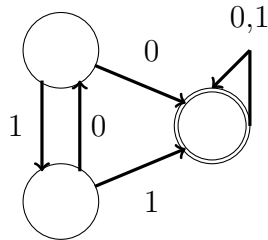
Homework 3

Mathematics in Computer Science

1. Classify the following sets as countably infinite or not countably infinite.
 - (a) the set of all integers
 - (b) the set of all pairs of integers
 - (c) all finite subsets of integers
 - (d) all subsets of integers
 - (e) the set of all reals
 - (f) all finite subsets of reals
 - (g) all subsets of reals
 - (h) the set of all finite length strings
 - (i) all finite subsets of the set of all finite length strings
 - (j) all subsets of the set of all finite length strings
 - (k) all computer programs
 - (l) functions mapping integers to the set $\{0, 1\}$
 - (m) functions mapping integers to integers
 - (n) the set of all random infinite length strings
 - (o) the set of all non random infinite length strings
2. Write out a careful proof that $\sum_{i=1}^n i = \frac{n(n+1)}{2}$. The problem will be graded on penmanship, clarity, and style of your solution.
3. Prove by induction that
 - (a) $(x + y)^n = x^n + \binom{n}{1}x^{n-1}y + \dots$
 - (b) $\sum_{i=1}^k (2i - 1) = k^2$
 - (c) $\sum_{i=1}^n i(i + 1) = 1 \times 2 + 2 \times 3 + 3 \times 4 + \dots + n(n + 1) = \frac{1}{3}n(n + 1)(n + 2)$
 - (d) $\sum_{i=1}^n (2i - 1)^2 = \frac{2}{3}n(2n + 1)(2n - 1)$
 - (e) Any $2^n \times 2^n$ checker board can be covered with tiles from the set below except for (i) the upper left hand corner (ii) except for any designated square. In each case clearly state your inductive hypothesis.



- (f) Prove that the automaton below accepts the set of all strings of 0's and 1's that either start with a 1 or have two consecutive 0's or two consecutive 1's.



- (g) The following two statements about a graph are equivalent
- i. For any three nodes x, y , and z if there is an edge from x to y and an edge from y to z , there is an edge from x to z .
 - ii. If there is a path from x to y , there is a direct edge from x to y .
- (h) Any finite game has winning strategy for either player 1 or player 2. Example tic tac toe
4. Given a zero knowledge proof that a graph has a Hamilton circuit. Hint: Encrypt things so the other person can ask for either the graph or the Hamilton circuit.