Homework 1

Mathematics in Computer Science

1. What are the sets
   (a) $\{0^i1 | i \in N\}$
   (b) $S_1 = \{101\}\{0^i10^{i+1}1 | i \in N\}^*$
   (c) $S_2 = \{1\}\{0^i10^{i+1}1 | i \in N\}^*\{0\}^*\{1\}$
   (d) $S_1 \cap S_2$
2. Create an exercise that illustrates the notion of closure for a class of sets other than these included in the class.
3. Which of the following classes of sets are closed under each of the following operations: union, intersection, power set operation? Explain your answer.
   (a) the class of all finite sets
   (b) the class of all infinite sets
   (c) the class of all sets of even cardinality
   (d) the class of all sets of odd cardinality
4. Prove that $S = (S \cap T) \cup (S - T)$
5. Write a formula for $|S_1 \cup S_2 \cup S_3|$ in terms of $|S_1|, |S_2|, |S_3|, |S_1 \cap S_2|, |S_1 \cap S_3|, |S_2 \cap S_3|, |S_1 \cap S_2 \cap S_3|$. Prove that your formula is correct.
6. Consider the non negative integers $N = \{0, 1, 2, \ldots\}$ and define $i \equiv j$ if remainder of i divided by 5 equals the remainder of j divided by 5.
   (a) Is the relation reflexive, symmetric and transitive?
   (b) What are the equivalence classes?
   (c) Select the smallest integer in each equivalence class to represent the equivalence class. Let $(i)_{mod}$ be the representative for the class containing i. Prove the following statements. $(i)_{mod}+(j)_{mod} = (i+j)_{mod}$ and $(i)_{mod}\times(j)_{mod} = (i\times j)_{mod}$.
   (d) Which are valid statements?
      i. $(5)_{mod} = (10)_{mod}$
      ii. $(6)_{mod} = (7)_{mod}$
   (e) Write out the addition and multiplication tables for the representative elements of the equivalence classes so that $(i)_{mod} + (j)_{mod} = (i + j)_{mod}$ and $(i)_{mod} \times (j)_{mod} = (i + j)_{mod}$
   (f) How can one add negative numbers to this system? That is, replace N by Z.

7. List three groups besides $(Z, +)$.

8. Consider the set of all two by two matrices of real numbers and the operation of multiplication given by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{pmatrix}$$

   Does the set of 2×2 matrices whose elements are real numbers form a group under multiplication? Is it commutative?

9. Prove that $S^* = \bigcup_{i=0}^{\infty} S^i$ is the smallest set containing $S \cup \{\epsilon\}$ and closed under concatenation.